Use of Biased-Relay Feedback for System Identification

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Since chemical processes are very complex and some parameters are often unknown or time varying, the derivation of rigorous dynamic model is very difficult and requires extensive engineering manpower. On the other hand, a good dynamic process model is always necessary for the design of the control system. Therefore, system identification becomes an important issue. Recent advances in stochastic or deterministic identification lead to a general conclusion: identify the model for the purpose of control system design (control relevant identification). However, interconnection characteristics of chemical processes impose another constraint: the identification procedure should give as little process upset as possible. In other words, closed-loop identification is preferred for chemical processes.

Recently, the relay feedback tests have received a great deal of attention in system identification. Åström and Hägglund (1984) suggest the use of an ideal (on-off) relay to generate sustained oscillation in closed-loop identification. Subsequently, the important process information, ultimate gain (K_u) and ultimate frequency (ω_u), can be found in a straightforward manner. The relay feedback system of an autotuner gains widespread acceptance in process industries for its reliability and simplicity (Hägglund and Åström, 1991; Wu et al., 1994) and many commercial products for autotuning have appeared in the market since the mid-1980s. Extensions of relay feedback system to monitoring and gain scheduling have also been made (Chiang and Yu, 1993; Lin and Yu, 1993; Luyben and Eskinat, 1994). Moreover, multivariable autotuners were also proposed (Shen and Yu, 1994; Friman and Waller, 1994).

The success of the relay feedback autotuner lies on the fact that it identifies *one* important point on the Nyquist curve: the point at the crossover frequency (ultimate frequency). Luyben (1987) is among the first to employ the relay feedback test for system identification. The autotune variation (ATV) method back calculates system parameters from ultimate gain and ultimate frequency obtained from a relay feedback experiment. Since only process information K_u and ω_u are available, additional process information, such as steady-state gain, should be known *a priori* in order to fit a typical transfer function (such as a first-, second- or third-order plus dead time system). In order to alleviate this stringent requirement, Li et al. (1991) and Leva (1994) propose the use of *two* relay feedback tests to find *two* points on the Nyquist curve and the least-square method is employed to estimate the parameters of the transfer function. Therefore, the time required for plant test increases as the number of experiments doubled. Then, a question remains: do we really need *two* relay feedback experiments to identify *two* points on a Nyquist curve?

The biased relay offers some light along this direction. Oldenburger and Boyer (1962) try to eliminate sustained oscillation in some nonlinear elements by injecting an additional sinusoidal signal with substantially different frequency at the input of the nonlinear element. This type of system was analyzed using the concept of equivalent gain from a biased relay (Oldenburger and Boyer, 1962). Hang et al. (1993) attempt to overcome the inaccuracy in the estimate of K_u and ω_u in the face of static load disturbance by introducing an automatic bias into an ideal relay. Luyben and Eskinat (1994) adjust the relay height (use asymmetrical relay) to obtain symmetrical output response such that the parameters of a Hammerstein model can be found from this test. Tseng and Wu (1992) and Wu et al. (1994) try to find the steady-state gain of a transfer function by changing the setpoint while performing a relay feedback test. One way or another, the concept of biased relay appears in the literature to overcome various problems facing linear or nonlinear systems.

The purpose of this work is to devise a relay feedback experiment that can identify two points on the Nyquist curve from a single test at the frequencies $\omega = 0$ and $\omega = \omega_u$. An identification procedure is devised accordingly for this parameter identification. In this article the theory of equivalent gain is described, the biased relay feedback system is analyzed, and the potential problems are explored. Parametric system identification is discussed followed by the conclusion.

Theory

Symmetric oscillation: Åström-Hägglund autotuner

The Åström-Hägglund autotuner (Åström and Hägglund, 1984) is based on the concept that when the output (y) lags

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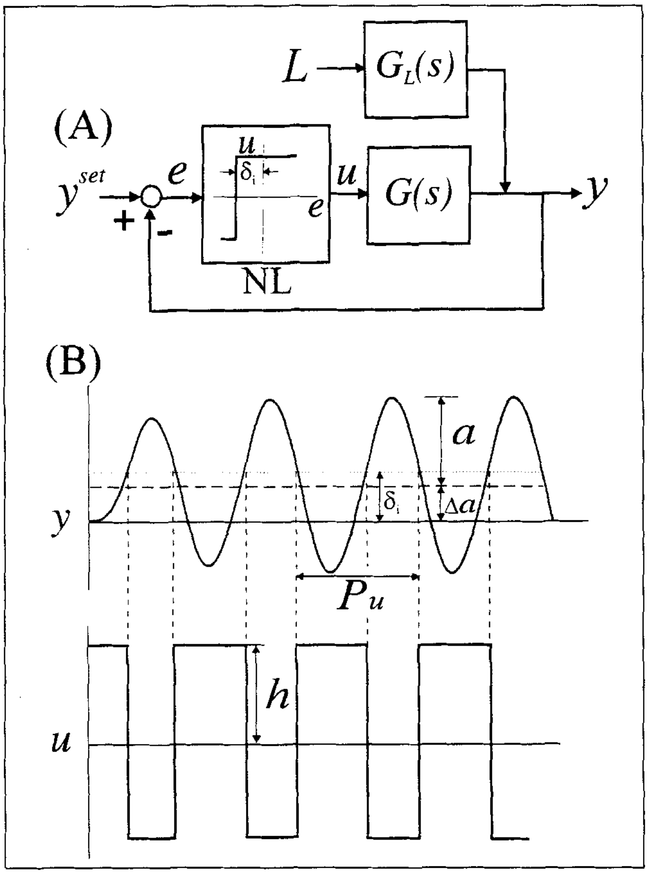

Figure 1. Input-biased relay feedback system.

(A) block diagram; (b) system input and output responses.

behind the input (u) by $-\pi$ rad. in closed-loop identification, the system output may oscillate with a period P_u. The ultimate gain (K_u) and ultimate frequency (ω_u) can be found from the principal harmonic approximation

$$K_u = \frac{4h}{\pi a} \tag{1}$$

$$\omega_u = \frac{P_u}{2\pi} \tag{2}$$

where a is the amplitude of the oscillation and P_u is the period of oscillation. Conventionally, an on-off relay is placed in the feedback loop to generate a symmetric limit cycle.

Asymmetric oscillations in feedback system

Considering a feedback system, $G(s)$ is a linear transfer function, NL is a nonlinear element and L is an external load, as shown in Figure 1. The following condition will lead the output y to produce an asymmetric limit cycle: (1) NL is an asymmetric nonlinearity (e.g., biased relay) or (2) L or y^{set} is a nonzero constant value. In either case, the input to the nonlinear element ($e(t)$) can be described as

$$e(t) = a \sin \omega t + \Delta a \tag{3}$$

where a is the magnitude of a sinusoidal wave and Δa is the magnitude of biased signal. Equation 3 shows that $e(t)$ consists of two parts: a sinusoidal wave and a biased signal. This is the well-known Dual-Input Describing Function (DIDF) (West et al., 1956). The output of the nonlinear element [$u(t)$] can be expressed in terms of Fourier series

$$u(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \tag{4}$$

where

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} u(t) d\omega t \tag{5}$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} u(t) \cos n\omega t d\omega t \tag{6}$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} u(t) \sin n\omega t d\omega t \tag{7}$$

Since only the principal harmonic is employed in linear analysis, the high-order terms ($n \geq 2$) from Fourier expansion are truncated for the purpose of analysis. Therefore, the linear approximation to the output of the nonlinear element can be expressed as

$$\begin{aligned} u(t) &= A_0 + A_1 \cos \omega t + B_1 \sin \omega t \\ &= \hat{u}_0 + \hat{u}_1 \sin(\omega t + \phi) \end{aligned} \tag{8}$$

where

$$\hat{u}_0 = A_0$$

$$\hat{u}_1 = \sqrt{A_1^2 + B_1^2}$$

$$\phi = \tan^{-1}\left(\frac{A_1}{B_1}\right)$$

In order to describe the characteristic of NL, the relationship of input and output signals of nonlinear element can be separated into two parts: one is the oscillatory part (the gain of sinusoidal wave to the output of nonlinear element) and another one is the static part (the gain of biased signal to the output of nonlinear element). Therefore, the DIDF are defined with respect to these two components

$$N_\gamma = \frac{\hat{u}_0}{\Delta a} \tag{9}$$

$$N = \frac{\hat{u}_1}{a} e^{j\phi} \tag{10}$$

both of which are functions of Δa and a (dual input). Generally, the oscillatory describing function (N) (such as $A_1 = 0$ as shown in Eq. 23) and the static describing function (N_γ) are real numbers. Furthermore, the following conditions should be satisfied for the existence of sustained oscillation (Cook, 1986).

$$1 + N_\gamma G(0) = 0 \quad (11)$$

$$1 + NG(j\omega) = 0 \quad (12)$$

Notice that the conditions for DIDF (Eqs. 11 and 12) differ from the condition for single-input describing function (SIDF) (Eq. 12). Here, the balance is achieved for two components: one is the static balance in the low-frequency element (Eq. 11) and another is the oscillatory condition in the high-frequency element (Eq. 12). This additional condition (Eq. 11) at zero frequency relates N_γ to the steady-state gain of the process, and this can be useful in system identification.

Generating asymmetric oscillation: input-biased relay feedback

Consider a closed-loop system where a biased relay (Figure 1a) is inserted in the feedback loop. The input-biased relay is characterized by two parameters: a relay height h and a bias δ_i. Figure 1b shows that the system output y gives an asymmetric response and the magnitude of the bias is: Δa. Furthermore, the bias shown in *y is not* equal to the bias in the relay δ_i ($\Delta a \neq \delta_i$). As for the system input u, the input-biased relay gives oscillations with unequal half period (Figure 1b). Physically, the input-biased relay feedback test can be visualized as an ideal relay plus a step change (a step change embedded in a relay feedback test).

The input-output responses of the input-biased relay can be analyzed analytically. Consider an input-biased relay with corresponding input (e) and output (u) (Figure 1b). Assuming that the input to the nonlinear element is an ideal sinusoidal wave plus a bias

$$e(t) = a \sin \omega t + \Delta a \quad (13)$$

Then, u can be characterized by θ with

$$\theta = \sin^{-1}\left(\frac{\delta_i - \Delta a}{a}\right) \quad (14)$$

DIDF analysis for the static part (Eq. 11) can be utilized to find the steady-state gain (K_p)

$$K_p = G(0) = -\frac{1}{N_\gamma} \quad (15)$$

Substituting Eq. 9 into Eq. 15, we have

$$K_p = -\frac{\Delta a}{\hat{u}_0} \quad (16)$$

From the output response in Figure 1b, $\hat{u}_0$ becomes

$$\hat{u}_0 = \frac{1}{2\pi}\int_0^{2\pi} u(t)\,d\omega t = -\frac{2h}{\pi}\sin^{-1}\left(\frac{\delta_i - \Delta a}{a}\right) \quad (17)$$

Therefore, the describing function analysis gives

$$K_p = \frac{\Delta a}{\frac{2h}{\pi}\sin^{-1}\left(\frac{\delta_i - \Delta a}{a}\right)} \quad (18)$$

This expression for the steady-state gain comes directly from the analysis of DIDF. Notice that Eq. 18 shows that K_p can be computed by observing system responses (e.g., Figure 1b).

In many occasions, the input to the relay $e(t)$ is not of sine wave like and the describing function analysis (Eq. 18) can lead to erroneous results. Another approach to find K_p is to compute the equivalent gain (N_γ) *numerically*. That is (Oldenburger and Boyer, 1962)

$$N_\gamma = \frac{\int_0^{2\pi} u(t)\,d\omega t}{\int_0^{2\pi} e(t)\,d\omega t} \quad (19)$$

Therefore, K_p becomes

$$K_p = -\frac{\int_0^{2\pi} e(t)\,d\omega t}{\int_0^{2\pi} u(t)\,d\omega t} \quad (20)$$

The next example is used to illustrate the difference between these two approaches (Eq. 18 vs. Eq. 20).

Example 1: WB Column (Wood and Berry, 1973). This is a $R-V$ controlled column. The transfer function of the $x_D - R$ loop is

$$G(s) = \frac{12.8e^{-s}}{16.8s + 1}$$

An input-biased relay is inserted in the feedback loop with a relay height $h = 1$ and a biased value $\delta_i = 0.1$. An asymmetric limit cycle is generated with a magnitude $a = 0.7397$ and a bias $\Delta a = 0.0942$. From Eq. 18, the steady-state gain becomes $K_p = 18.87$. This describing function approximation overestimate K_p by a factor of 47.5%. The reason for that is the derivation of Eq. 18 is based on the assumption that the input signal to the nonlinear element is an ideal sinusoidal wave. In fact, the assumption does not hold for typical first-order plus dead time system (Chiang et al., 1992). An alternative is to compute K_p numerically according to Eq. 20. That gives

$$K_p = -\frac{\int_0^{2\pi} e(t)\,d\omega t}{\int_0^{2\pi} u(t)\,d\omega t} = 12.79$$

Obviously, this gives very accurate estimate of the steady-state gain (less than 1% error).

The example clearly indicates that very accurate estimate of low frequency information, steady-state gain, can be

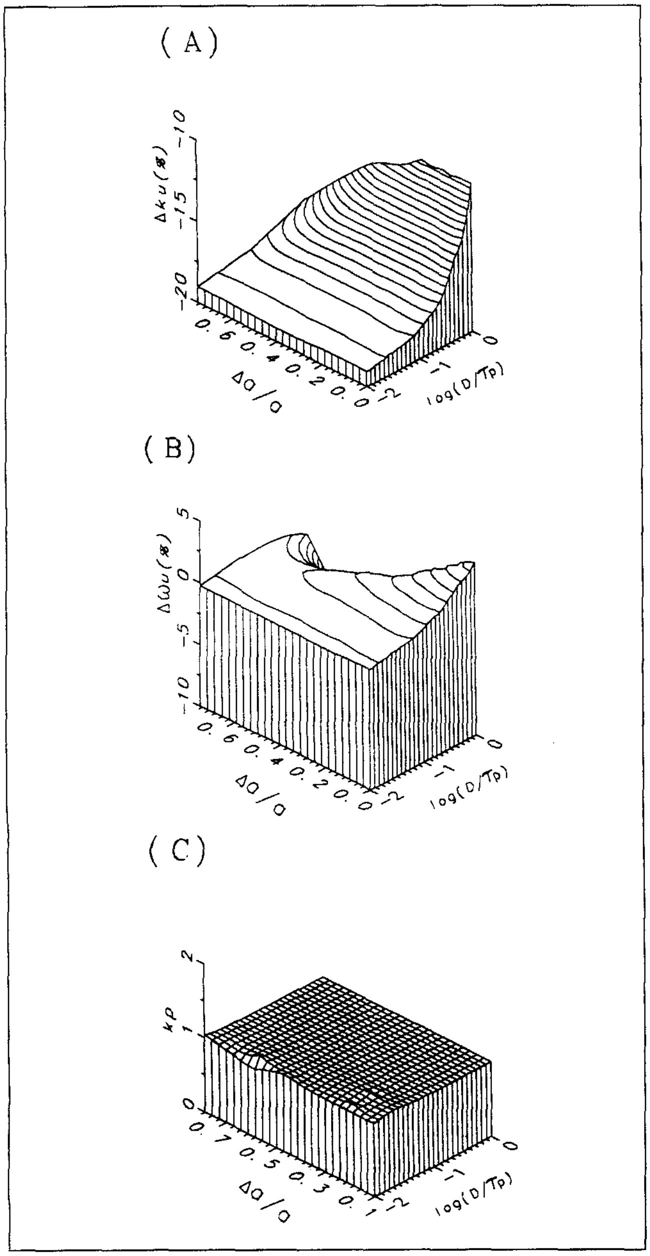

Figure 2. Input-biased relay feedback for first-order model with a range of system parameters.

(A) percent error in K_u; (B) percent error in ω_u; (C) estimated steady-state gain.

achieved using input-biased relay feedback. It should be noticed that the steady-state gain is obtained in addition to the ultimate gain and ultimate frequency observed from oscillatory input-output responses (Eqs. 1 and 2).

Discussions

Accuracy in K_u and ω_u

The additional information (K_p) obtained from biased relay feedback does not come without paying any price. The following examples illustrate a tradeoff between low frequency information and high frequency information. Consider a first-order plus dead time model.

$$G(s) = \frac{K_p e^{-Ds}}{\tau_p s + 1} \tag{21}$$

The following parametric spaces are studied: $K_p = 1$, $\tau_p = 10$ and $D/\tau_p = 0.01 \sim 1$. If an output-biased relay with different bias values ($\Delta a/a$ ranging from 0 (an ideal relay) to 0.6) is employed, the estimates of K_u, ω_u and K_p can be found from these relay feedback tests. The results (Figure 2) show that the estimate of ω_u deteriorates as the bias increases. However, the estimates of K_u and K_p remain fairly constant for different $\Delta a/a$'s as shown in Figure 2.

The result is not totally unexpected. Consider Fourier expansion of $u(t)$ (Eq. 4). The *nonzero* Fourier coefficients for the input-biased relay are

$$A_0 = -\frac{2h}{\pi}\sin^{-1}\left[\frac{\delta_i - \Delta a}{a}\right] \tag{22}$$

$$A_n = \frac{4h}{n\pi}\sin\left[n\sin^{-1}\left(\frac{\delta_i - \Delta a}{a}\right)\right], \quad n = 2, 4, 6, \ldots \tag{23}$$

$$B_n = \frac{4h}{n\pi}\cos\left[n\sin^{-1}\left(\frac{\delta_i - \Delta a}{a}\right)\right], \quad n = 1, 3, 5, \ldots, \tag{24}$$

which means that we have the following coefficients, A_0, A_2, A_4, A_6, ... and B_1, B_3, B_5, ..., are nonzero. However, for an unbiased (ideal) relay ($\delta_i = 0$ and $\Delta a = 0$) the *remaining* nonzero coefficients are: B_1, B_3, B_5, ... Since only principal harmonic, e.g., the term with the subscript 1, is involved in the describing function analysis, the truncation of higher-order terms in the input-biased relay can lead to a greater error in the estimate of K_u and ω_u (ω_u in particular, Figure 2b).

Measurement noise

Any practical method for on-line estimation should be able to overcome process noise. Here, the measurement noise is introduced to the process to test the effectiveness of biased relay feedback system. Note that a relay with hysteresis can be used to overcome process noises (Chiang and Yu, 1993). The $x_D - R$ element of WB column example is used to illustrate the effect of process noises and the steady-state gain is the major concern.

$$G(s) = \frac{x_D}{R} = \frac{12.8e^{-s}}{16.8s + 1}$$

For the WB column without measurement noise, the input and output biased relays are employed to find the steady-state gain. Here, the relay height is 1 ($h = 1$) and the input bias is 0.1 ($\delta_i = 0.1$) is used. The results show that the steady-state gain, ultimate gain and ultimate frequency are: $K_p = 12.792$, $K_u = 1.72$, $\omega_u = 1.616$. In computing K_p, 3 periods of oscillation are employed in the integration. Next, a measurement noise is introduced to x_D. Assume the noise is zero mean and Gaussian distributed with the noise to signal ratio of 0.02 ($N/S = 0.02$). Figure 3 shows the input-output responses for the input-biased relay feedback test. The result of the steady-state gain calculation as we change the number of pe-

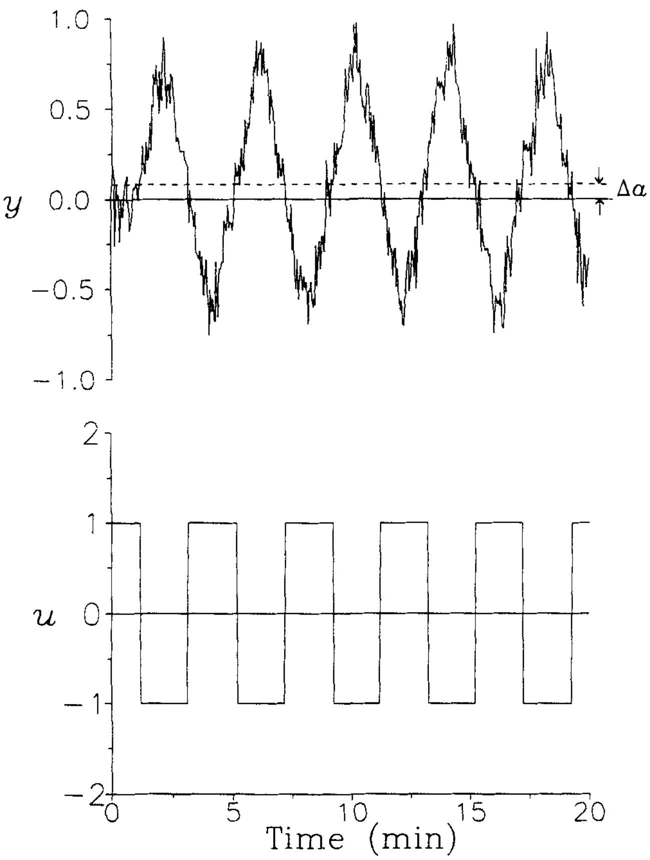

Figure 3. Input-biased relay feedback test for WB column corrupted with measurement noise.

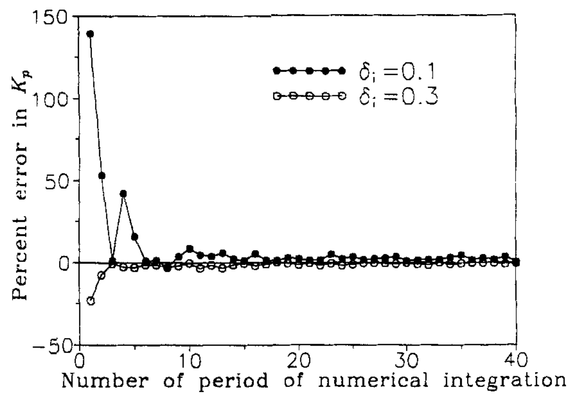

Figure 4. Estimate of steady-state gain as a function of number of period used in numerical integration for WB column corrupted with measurement noise.

riods used in the integration (Eq. 20) are shown in Figure 4. Figure 4 clearly indicates that significant error may result unless a much longer period of integration is used. For example, if the number of period of integration is 10, then the steady-state gains obtained is 15.7. Its implication is that a much longer time is needed for the relay feedback experiment in order to have a better estimate of K_p. One way to overcome this problem is to increase the biased value (δ_i). Figure 4 shows that if $\delta_i = 0.3$, a much shorter time period is required to find an accurate estimate of K_p.

Nonlinearity

For nonlinear chemical processes, the system may exhibit asymmetric oscillation under an ideal relay feedback test. This may result in significant error in estimating the steady-state gain (K_p) under the biased relay feedback. The reason is that K_p is computed from the ratio of the net areas from system input-output responses and the net areas are obtained from the asymmetry. Here, both the input-biased relay and process nonlinearity contribute to the asymmetric oscillation. The following example is used to illustrate the potential problem.

Example 2: Nonlinear System. Consider a Hammerstein model (Luyben and Eskinat, 1994), the linear part transfer function is

$$G(s) = \frac{y}{x} = \frac{e^{-s}}{(5s+1)(s+1)} \tag{25}$$

and the nonlinear part is

$$x = 2[1 - \exp(-0.693u)] \tag{26}$$

The linear relationship between y and u of the Hammerstein model can be found by linearizing the nonlinear part (Eq. 26), that gives

$$\tilde{G}(s) = \frac{1.386e^{-s}}{(5s+1)(s+1)} \tag{27}$$

An ideal relay feedback with $h = 1$ results in asymmetry in the output responses (Figure 5 at $t = 0 \sim 23$ min). Moreover, the estimate of K_u is off by a factor of 27.6% (3.712 as opposed to the exact value of 5.126). Next, an input-biased relay with $\delta_i = 0.3$ is inserted in the feedback loop, and the steady state obtained from numerical integration is 0.293 (off by 78.9%). If we decrease δ_i further to 0.1, the resultant K_p becomes negative ($K_p = -0.1214$).

An approach to overcome this problem is to use an output-biased relay (Wu, 1994) (relay with an asymmetry in the outputs: $h + \delta_o$ and $-h + \delta_o$). For the system with asymmetric output response, the output bias (δ_o) is adjusted such that the output oscillation (y) become symmetric as shown in Figure 5 ($t = 23 \sim 45$). After this adjustment is made, an input-biased relay feedback with $\delta_i = 0.1$ is performed on feedback loop to find K_p, K_u and ω_u. The results are $K_p = 1.267$, $K_u = 4.586$ and $\omega_u = 0.929$. Here, much better estimates of K_p, K_u and ω_u are obtained.

Ongoing analyses clearly indicate that along with the benefit of the biased relay feedback, it may deteriorate the estimates of K_u and ω_u as the biased value increases (Figure 2) or the estimate of K_p as the noise level increases. Therefore, care has to be taken in deciding the biased level. A simple heuristics is: make the biased as small as possible and yet provide enough process excitation to overcome process noises. Process nonlinearity can lead to the erroneous result in the estimate of steady-state gain. A procedure is proposed to overcome process nonlinearity. This two-step procedure establishes symmetric oscillation using an output-biased relay followed by an input-biased relay feedback to find K_p, K_u and ω_u.

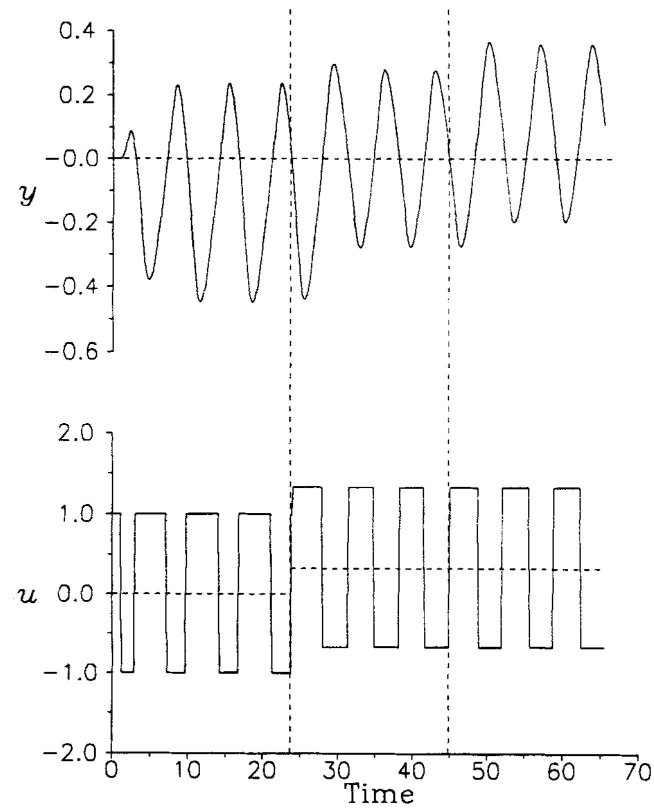

Figure 5. Biased relay feedback test for high nonlinear system.

System Identification

Luyben's (1987) original ATV method uses one relay feedback test to determine K_u and ω_u. However, this method requires that the steady-state gain (K_p) must be known to find the process transfer function. Attempts have been made to eliminate the stringent requirement: K_p has to be known *a priori* (Li et al., 1991; Leva, 1993). These attempts involve at least *two* relay feedback tests, one at ω_u and the other at a frequency with phase angle around $-130° \sim -140°$ (Li et al., 1991), followed by a least-square regression. The proposed biased relay feedback, on the other hand, involves only *one* relay feedback test to obtain process information at two frequencies: one at ultimate frequency and the other at *zero* frequency. The ultimate gain K_u and ultimate frequency ω_u are observed from biased relay feedback (e.g., Figure 1) (Eqs. 1 and 2). Moreover, the steady-state gain (K_p) can be found from integrating system input-output response (Eq. 20). One can utilize these (K_p, K_u and ω_u) to find the parameters of typical transfer functions. Two model structures are considered: first- and second-order plus dead time models.

(1) Model 1 (One Lag Plus Dead Time)

$$G(s) = \frac{K_p e^{-Ds}}{\tau_p s + 1} \tag{28}$$

Since K_p, K_u and ω_u are available, the following two equations are useful to find model parameters.

$$\frac{1}{K_u} = \frac{K_p}{\sqrt{1 + (\tau_p \omega_u)^2}} \tag{29}$$

$$-\pi = -\tan^{-1}(\tau_p \omega_u) - D\omega_u \tag{30}$$

Once K_p is known, the time constant τ_p can be calculated directly from Eq. 29. With τ_p available, the dead time D can be found from Eq. 30. Notice that for this model (Model 1), the proposed ATV method differs from the original ATV method in that the dead time is *calculated* instead of being observed. This can be useful since comparison can be used to *validate* the model structure. If the calculated D differs substantially from the observed D, the next model structure is explored.

(2) Model 2 (Two Unequal Lags Plus Dead Time)

$$G(s) = \frac{K_p e^{-Ds}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{31}$$

First, the dead time D is observed from output response. With K_p available, the following two equations are useful in parameters estimation

$$\frac{1}{K_u} = \frac{K_p}{\sqrt{[1 + (\omega_u \tau_1)^2][1 + (\omega_u \tau_2)^2]}} \tag{32}$$

$$-\pi = -\omega_u D + \tan^{-1}(-\omega_u \tau_1) + \tan^{-1}(-\omega_u \tau_2) \tag{33}$$

Since τ_1 and τ_2 are the only two unknown, Eqs. 32 and 33 can be solved simultaneously to find τ_1 and τ_2. Obviously, this method is not limited to first- or second-order model (e.g., Model 3 or 5 in Luyben (1987)). However, in general, the first- or second-order plus dead time transfer function will suffice for the purpose of control system design for most chemical processes.

Example 3. First-Order System (Li et al., 1991)

$$G(s) = \frac{e^{-2s}}{10s + 1}$$

First, an input-biased relay with $h = 1$ and $\delta_i = 0.05$ is employed in the feedback loop. Denote as b_i-ATV hereafter. Experimental results give $K_p = 0.999$, $K_u = 7.023$ and $\omega_u = 0.855$. If a first-order model structure is assumed, the time constant and dead time become: $\tau = 8.118$ and $D = 2.005$. However, if the model structure is chosen to be a second-order system, the proposed input-biased relay feedback gives: $\tau_1 = 8.118$ and $\tau_2 = 0.0035$. The results clearly indicate that this is effectively a first-order system ($\tau_2 \approx 0$).

Example 4: Second-Order System (Li et al., 1991)

$$G(s) = \frac{e^{-2s}}{(10s + 1)(s + 1)}$$

An input-biased relay with $h = 1$ and $\delta_i = 0.05$ is used in the feedback loop. A relay feedback test gives: $K_p = 0.998$, $K_u = 6.553$ and $\omega_u = 0.597$. From Eqs. 32 and 33, the two time constants are: $\tau_1 = 9.14$ and $\tau_2 = 1.044$. If the model structure is selected as a first-order system, the time constant and dead time (calculated from Eqs. 29 and 30) are 10.82 and 2.00, respectively. The results again illustrate that, despite the fact that an incorrect model structure is selected, b_i-ATV is quite reliable in the estimation of model parameters.

The results clearly show that the proposed ATV methods are effective and reliable in parametric system identification. More importantly, the results are obtained using only *one* relay feedback test.

Conclusion

Based on the concept of dual-input describing function DIDF, the input-biased relay feedback experiments are proposed for system identification. In addition to the critical point (K_u and ω_u), the steady-state gain (K_p) can also be found in a single relay feedback test. Describing function analyses of the biased relay feedback are given and extensions to system identification are proposed. Moreover, potential problems associated with the biased relay feedback are discussed and suggestions to avoid them are also given. For a process corrupted with measurement noises, increasing the biased value (δ_i) will shorten the number of integration periods to find an accurate estimate of K_p. A two-step biased relay feedback test is proposed to overcome the process nonlinearity in the estimate of steady-state gain. Finally, the ATV method is proposed for the identification of parametric transfer functions. Two linear examples are used to test the effectiveness of the proposed method. Results show that the biased relay feedback gives satisfactory performance under realistic process environment.

Notation

A_o = Fourier coefficient of constant term
A_n = Fourier coefficient of sine wave term
ATV = autotune variation
B_n = Fourier coefficient of cosine wave term
b_i-ATV = input-biased ATV
D = time delay
h = magnitude of relay output
s = Laplace transform variable
SIDF = single-input describing function
$\hat{u}_0$ = static output magnitude of nonlinear element
$\hat{u}_1$ = principal harmonic output magnitude of nonlinear element

Greek letters

δ_i = bias in the input to nonlinear element
δ_o = bias in the output to nonlinear element
ϕ = phase shift in the output of nonlinear element

Literature Cited

Åström, K. J., and T. Hägglund, "Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins," *Automatica*, **20**, 645 (1984).
Alatiqi, I. M., and W. L. Luyben, "Control of a Complex Sidestream Column/Stripper Distillation Configuration," *Ind. Eng. Chem. Process Des. Dev.*, **25**, 762 (1986).
Chiang, R. C., S. H. Shen, and C. C. Yu, "Derivation of Transfer Function from Relay Feedback Systems," *Ind. Eng. Chem. Res.*, **31**, 855 (1992).
Chiang, R. C., and C. C. Yu, "Monitoring Procedure for Intelligent Control: On-Line Identification of Maximum Closed-Loop Log Modulus," *Ind. Eng. Chem. Res.*, **32**, 90 (1993).
Cook, P. A., *Nonlinear Dynamical Systems*, Prentice-Hall, New York (1986).
Eskinat, E., S. H. Johnson, and W. L. Luyben, "Use of Auxiliary Information in System Identification," *Ind. Eng. Chem. Res.*, **32**, 1981 (1993).
Friman, M., and K. Waller, "Autotuning of Multiloop Control Systems," *Ind. Eng. Chem. Res.*, **33**, 1708 (1994).
Hang, C. C., K. J. Åström, and W. K. Ho, "Relay-Auto-Tuning in the Presence of Static Load Disturbance," *Automatica*, **29**, 563 (1993).
Hägglund, T., and K. J. Åström, "Industrial Adaptive Controllers Based on Frequency Response Techniques," *Automatica*, **27**, 599 (1991).
Leva, A., "PID Autotuning Algorithm Based on Relay Feedback," *IEE Proc.*, **140D**, 328 (1993).
Li, W., E. Eskinat, and W. L. Luyben, "An Improved Autotune Identification Method," *Ind. Eng. Chem. Res.*, **30**, 1530 (1991).
Lin, J. Y., and C. C. Yu, "Automatic Tuning and Gain Scheduling for pH Control," *Chem. Eng. Sci.*, **48**, 3159 (1993).
Luyben, W. L., "Derivation of Transfer Functions for Highly Nonlinear Distillation Columns," *Ind. Eng. Chem. Res.*, **26**, 2490 (1987).
Luyben, W. L., *Process Modeling, Simulation and Control for Chemical Engineers*, McGraw-Hill, Singapore (1990).
Luyben, W. L., and E. Eskinat, "Nonlinear Auto-Tune Identification," *Ind. J. Control*, **59**, 595 (1994).
Oldenburger, R., and R. C. Boyer, "Effects of Extra Sinusoidal Input to Nonlinear Systems," *Trans. ASME*, **84D**, 559 (1962).
Shen, S. H., and C. C. Yu, "Use of Relay-Feedback Test for Automatic Tuning of Multivariable Systems," *AIChE J.*, **40**, 627 (1994).
Tseng, C. G., and W. T. Wu, "Robust Control Based on an Identified Model via a Modified Autotune Variation Method," *J. CIChE*, **23**, 227 (1992).
West, J. C., J. L. Douce, and R. K. Livesley, "The Dual-Input Describing Function and Its Use in the Analysis of Non-linear Feedback Systems," *Proc. IEE*, **103B**, 463 (1956).
Wood, R. K., and M. W. Berry, "Terminal Composition Control of a Binary Distillation Column," *Chem. Eng. Sci.*, **28**, 707 (1973).
Wu, C. L., C. K. Kao, H. S. Chou, S. J. Ho, and H. Y. Yuang, "Theory and Applications of PID Autotuners," *Symposium on Computer Process Control*, 238, Taipei (1994).
Wu, J. S., "Use of Biased-Relay Feedback Test in System Identification," MS Thesis, National Taiwan Institute of Technology, Taipei (1994).
Wu, W. T., C. G. Tseng, and Y. T. Chu, "System Identification and On-Line Robust Control of a Multivariable System," *Int. J. Syst. Sci.*, **25**, 423 (1994).

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